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Effects of string tension and impact location on tennis playing [†]

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Abstract

Finite element impact simulations were performed to observe the vibration of a tennis racket and its strings, as well as the effects of string tension and impact location on a player's hand and his chances of getting an injury. Studies using the finite element method [FEM] revealed that decreasing the string tension lowers the coefficient of restitution. The ratio of speed to angle change increases with a decrease in string tension. Moreover, the resultant force on the player's hand is stronger if the tennis ball hits the dead spot than if it hits the sweet spot. For instance, as a tennis ball hits the dead spot with a speed of 10.05m/s, an angle of 15°, and a string tension of 222N, the player's hand feels a maximum resultant force of almost 424N, which is 1.61 times higher than if the ball hits the sweet spot, at t=0.081 and t=0.0149. Moreover, the force exerted on the player's hand if the ball hits either the best-bounce spot or the off-center spot is 1.4 times higher than if the ball hits the sweet spot.

Keywords: Tennis racket; Finite element analysis; Vibration; String tension; Resultant force

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1. Introduction

With the recent development of new materials and technologies in sports engineering, the application of engineering mechanics has been extensively employed to optimize the design of sports goods such as tennis rackets. The key factors that need to be considered in manufacturing sports equipment, especially tennis rackets, are precision (control), power, ergonomics, and injury prevention [1].

With regard to elbow injury which is a common occurrence in tennis, the low-frequency vibrations related to hitting a ball with a racket have an adverse effect on human joints. As a consequence, a player feels pain in the elbow. Elbow injury, also called "tennis elbow", is the weakening of the extensor muscle in the elbow. Besides low-frequency vibrations, stresses caused by simultaneous twisting and bending while hitting the ball also cause tennis elbow. This injury takes time to heal, thus affecting not only a tennis player's performance in court but also his ability to do other activities. Therefore, analysis of a tennis racket and identification of the causes of tennis elbow are very important as far as a tennis player is concerned [1].

To better understand the implications of a tennis ball's impact on a racket and the vibration behaviors of a tennis racket, numerous studies on forehand stroke have been performed following the pioneering work of Casolo, Lorenzi, and Lucchini [2]. They studied the relationship among string tension, ball speed, and coefficient of restitution. They clarified that an increase either in string tension or ball speed results in greater energy dissipation mainly due to ball deformation. Buechler [3] studied the effect of the size and location of a sweet spot on the level of vibration of a tennis racket. Over the past 30 years, the size of the sweet spot in tennis rackets had been expanding as designers understood the need to reduce the vibration felt by a tennis player every time a tennis ball would hit the racket he was holding [3]. Vic Braden and Howard Brody [4] said that the sweet

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spot was not fixed but moved depending on the radius of the swing. Cross's experimental studies [5, 6] aimed at understanding the effects of hand force exerted on the center of percussion (COP) showed that a player feels best when the impact point is at or near the vibration nodes of the first few modes. He feels worse if the impact point is either the free-supported racket COP or the hand-held racket COP. Moreover, Bower, [7, 8] showed in their study of string tension effect under laboratory conditions that high string tension produced lower rebound angles and could contribute to the greater number of net errors, whereas low string tension was more likely to provide greater rebound velocity and allow longer traveling of the ball. Zhuang and Chen [9] studied the effects of boundary condition (hand-grasp, free-free, fixed-free) on the vibration of a racket. Recently, Gu and Li [10] used the FEM to study the dynamic characteristic of a tennis racket and string, and to investigate the effect of string tension on the modal frequencies and the shape of each mode. It was found out that an increase in string tension decreases the displacement of the racket and the frequencies by almost 70 percent, thus raising the tennis player's chances of having tennis elbow.

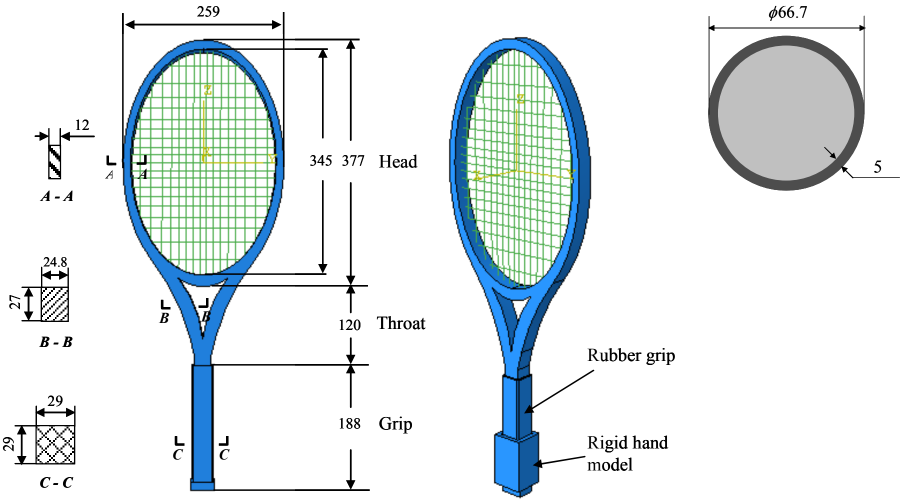
Nonetheless, the focus of previous studies was more on power and precision. Tennis elbow prevention and ergonomics have not received much attention despite injuries suffered by tennis players.

In this study, computer simulation techniques based on the FEM were used to observe the effects, under a forehand stroke, of the vibration of a tennis racket resulting from its collision with a tennis ball and of the resultant reaction force on the hand. In addition, the effects of string tension and impact location on the deformation of a tennis racket and on a player's hand, as well as their ability to cause elbow injury are investigated.

2. Method

2.1 Model and properties

Tennis racket modeling was performed using CATIA V5 and included the frame, strings, and rubber grip. Meshing was performed using HyperMesh7.0. The simulation was completed using ABAQUS 6.5-1 explicit [11]. Two kinds of boundary condition for gripping were used for the vibration mode analysis of the frame: without a rubber grip, and with a rubber grip and a rigid hand. The model without a rubber grip – which has a head length of 377 mm, head width of 259 mm, throat length of 120 mm, throat width of 24.8 mm, grip length of 188 mm, and grip width of 29 mm – is shown in Fig. 1(a). The model with a rubber grip and a rigid hand is shown in Figure 1b. The cross-sectional frame and its dimensions are shown in the left picture of Figure 1a. Figure 1c shows the model of a tennis ball, which has a diameter of 66.7 mm and a rubber skin thickness of 5 mm. The whole process was simulated based on



(a) (b) (c)

Fig. 1. (a) Model without a rubber grip (b) Model with a rubber grip and rigid hand (c) Model of ball (dimensions are expressed in millimeters).

0.03 second.

The frame of the tennis racket was modeled using 6075 solid elements to represent carbon fiber material with the following properties: Young's modulus of 25000 MPa, Poisson ratio of 0.3, and density of 1750 kg/m³. The strings were modeled as linear elastic using 626 T3D2 truss elements made of nylon with the following properties: Young's modules of 6895 MPa, Poisson ratio of 0.25, density of 1068 kg/m³, and cross-sectional area of 1.43 mm². The strings' initial tension was given as the stresses in the keyword of INITIAL CONDITIONS [11]. The tennis ball was modeled as a sphere with 458 S4R shell elements made of rubber to represent the Mooney-Rivlin material [12] with the constants C_{10}=0.69 MPa and C_{01}=0.173 MPa. Since ABAQUS/Explicit also requires some compressibility for hyper elastic materials, D_1=0.0145MPa^{-1} and a density of 1068 kg/m³ were chosen. The ball was subjected to an initial internal pressure of 0.041 MPa in addition to an ambient pressure of 0.1 MPa. The rubber grip was modeled as 320 solid elements, and its properties were assumed to be the same as those of the ball. The thickness of rubber grip was 3 mm. The hand was also assumed to be rigid and modeled as 200 solid elements.

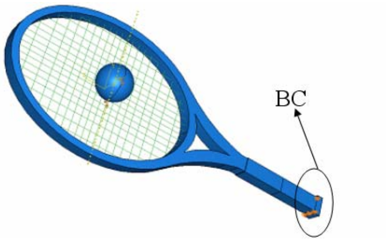


Fig. 2. Boundary condition for string tension analysis.

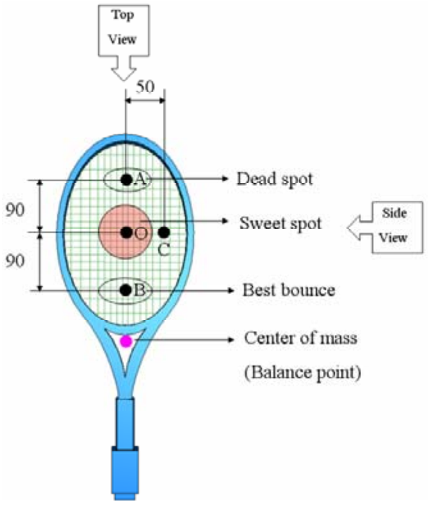


Fig. 3. Different positions of the impact point.

2.2 Simulation

2.2.1 String tension analysis

The first simulation examined the influence of the string tension on control and power in the case of a tennis racket without a rubber grip as shown in Fig. 1(a). Only one racket mode was used with five different string tensions: 177N (40 lbs), 222N (50 lbs), 266N (60 lbs), 311N (70 lbs), and 355N (80 lbs). The pre-impact velocity of the ball hitting the sweet spot was 10.05m/s at a 15° angle from the vertical center of the frame (angle of incidence) [1], and the bottom grip firmness was taken as the boundary condition (BC) as shown in Fig. 2.

2.2.2 Impact location analysis

In the second simulation, the impact location of the ball on the strings was adjusted to determine the location where the minimum shock or jar was felt by the hand.

In Fig. 3, O is described as the sweet spot, B the best-bounce spot, and A the dead spot. C was selected to determine the effect of an off-center impact on the twisting feel of the hand. The sweet spot is commonly located on the longitudinal axis between the tip and the throat, and the size of the sweet spot is known to be affected by the string tension [1]. The sweet spot is the COP, where the translational and rotational forces cancel each other, resulting in a minimal sensation of hitting the ball if the corresponding axis of rotation passes through the hand. The sweet spot is also considered the impact point of the maximum coefficient of restitution (COR) on the racket head, which gives the maximum ball speed and power (rebound). The sweet spot is also the nodal point of zero displacement in the vibration mode shape of the racket. The sweet spot serves an important role in tennis performance and is, therefore, often identified by tennis racket manufacturers.

On the other hand, the dead spot is the point where the ball does not bounce. This may be found when the racket handle is clamped to the top of a table, or pressed by a hand on the top of a table, allowing the rest of the racket to hang over the edge of the table, and a ball is dropped onto the strings.


(a) Free B.C (b) Fixed B.C (c) Hand grasp B.C

Fig. 4. Three boundary conditions for vibration analysis.

Table 1. Effects of string tension on power and impact characteristics.

	String tension				
	177N	222N	266N	311N	355N
Duration of ball-string Contact	4.655 E-03s	4.575 E-03s	4.655 E-03s	4.125 E-03s	4.055 E-03s
Post-impact speed as a % of pre-impact speed	76.4%	76.2%	75.8%	74.6%	74.3%
COR	0.773	0.769	0.763	0.751	0.749
Ratio of speed angle change	0.217	0.147	0.109	0.108	0.101

All the tests were conducted for the tennis racket with a rubber grip and a rigid hand as shown in Fig. 1(b). A string tension was fixed at 222N, and the friction coefficients between the surfaces of the rubber and the frame, and the hand and the grip were given as 0.1, respectively.

2.2.3 Vibration analysis

When the ball impacts on the strings, the local tension of the string around the impact region can increase to several hundred pounds. This causes the racket to deform and the frame to snap back, overshoot its equilibrium configuration, and oscillate for a period of time, depending on how it is damped. Thus, a finite element model of a tennis racket without a rubber grip, as shown in Figure 1a, was considered to simulate the free vibrational mode and its dependency on boundary condition of the grip. Three boundary conditions (Fig. 4) were considered in investigating the differences in vibration: (a) free boundary condition (freely supported), (b) fixed boundary condition (firm bottom grip, no displacement, no rotation), and (c) hand grasp boundary condition (half-firm grip, no displacement, possibility of rotation).

3. Results and discussion

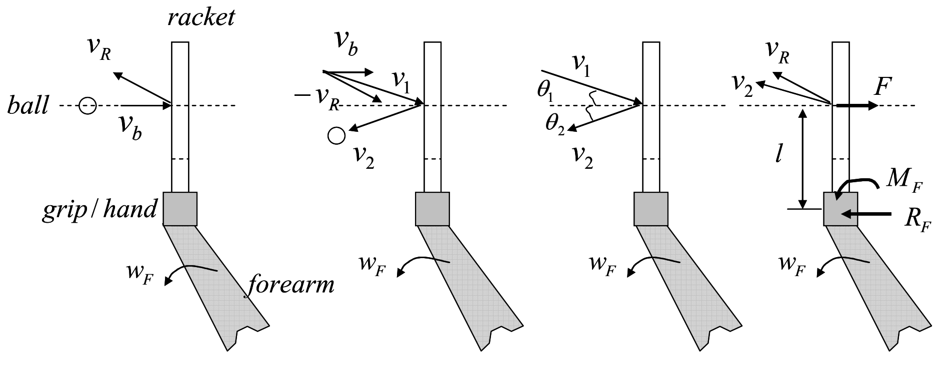
3.1 String tension analysis

Table 1 shows the numerical results when the tennis ball collides with the sweet spot with a speed of 10.05m/s and an angle of 15°. It is clear that increasing the string tension reduces the duration of the ball-string contact and the ratio of the post-impact speed to the pre-impact speed. Speed is described as the vector sum of the velocity components of the finite element nodes consisting the ball. A decrease in the string tension results in the reduction of the coefficient of restitution, which is the ratio of the vertical ball speed-out (rebound) to the vertical ball speed-in (incident) [1]. The larger the COR ratio, the more powerful the racket is. In any collision, some energy is lost to vibration and friction. The ball compresses as it hits the strings. The rubber stores some of the produced energy, which is then released as the ball becomes uncompressed.

Fig. 5 shows the schematic view of the rebound of a tennis ball when the forearm and the racket rotate at an angular velocity w. In this study, the ball is assumed to be incident from the left at speed v_b on a racket moving at speed v_R at a horizontal angle as shown in Fig. 5(a). This impact can be viewed in a racket frame of reference (sometimes called relative frame of reference), where the racket is at rest, by subtracting the v_R vector as shown in Fig. 5(b). If the ball is relatively incident at speed v_1 in the racket frame, it will rebound at speed v_2 as shown in Figs. 5(b) and 5(c). Fig. 5(d) shows the outcome in the court frame. This collision will exert a force F on the racket and yield a reaction force F_w and a bending moment $M_F (= Fl)$ on the player's hand, simultaneously.

Table 1 shows the ratio of the speed angle change measured in the racket frame, defined as $(\theta_1-\theta_2)/\theta_1$, where θ_1 (=15°) and θ_2 are the angles between normal axis to the racket frame and incident velocity vector v_1 , and rebound velocity vector v_2, respectively. The ratio of the speed angle change increases with a lower string tension. This means that a lower string tension allows the ball to rebound closer to the racket face.

Thus, a high string tension produces more deformation of the ball and releases more energy. The elastic deformation of a tennis ball during impact is shown by the magnified view presented in Figure 6, which particularly shows the effects of a string tension that measures 177N.



(a) (b) (c) (d)

Fig. 5. (a) Court frame of reference (b) Racket frame of reference (c) Explanation of θ_1 and θ_2 (d) Forces exerted on the racket and the player's hand in court frame of reference.

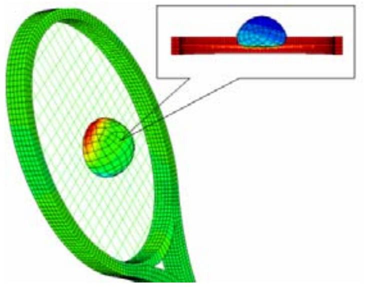


Fig. 6. Snapshot of the most extreme ball compression during impact.

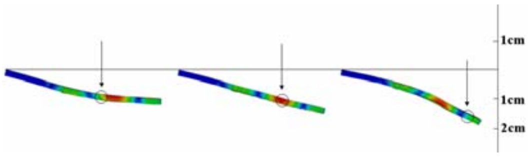


Fig. 7. Snapshots of the maximum displacement of rackets at different impact points.

3.2 Impact location analysis

Fig. 7 shows a side view of the racket and the maximum amplitude of the vibration for an impact time of around 12.0E-03s. The deformation magnified the simulation results by 10 times to show the clear difference in the amplitude. The size of the racket, and the values 1 cm and 2 cm are actual. In this figure, an off-center impact created more deformation.

The center of mass (CM) is usually in the throat of a racket (Fig. 3). Thus, if a ball strikes the racket well above this balance point, near the center of the head, the racket will recoil to conserve a linear momentum, while twisting or rotating about the CM to conserve an angular momentum. The particular ball-impact location on the strings, where the motions cancel at the hand, is the COP [1].

Table 2. Effects of the different impact points on power and deformation of a racket.

	Impact point			
	At O	At A	At B	At C
Duration of ball–string contact	4.955 E-03s	4.955 E-03s	4.955 E-03s	4.955 E-03s
Post-impact speed as a % of pre-impact speed	76.3%	67.7%	81.3%	71.3%
COR	0.764	0.669	0.818	0.722
Ratio of speed angle change	0.155	-0.138	0.120	0.233
Maximum displacement at impact points	1.64cm	1.92cm	1.26cm	1.58cm

Fig. 8 shows a vertical view of the racket and the ball-string impact at points O (center of head), C, and A as indicated in Fig. 3. The magnifying scale factor was 1.5, showing a clear difference in the deformation mode for an off-center impact.

A summary of the post-impact speed as a percentage of the pre-impact speed and the COR is shown in Table 2, where the impact location clearly affected the ball speed and power. The large COR also provided the high-return ball speed when a ball impacts on the position of best bounce at B.

Fig. 9 shows the variation in the resultant reaction (rebound) force on the player's hand transmitted through the racket during the time of impact. Here,

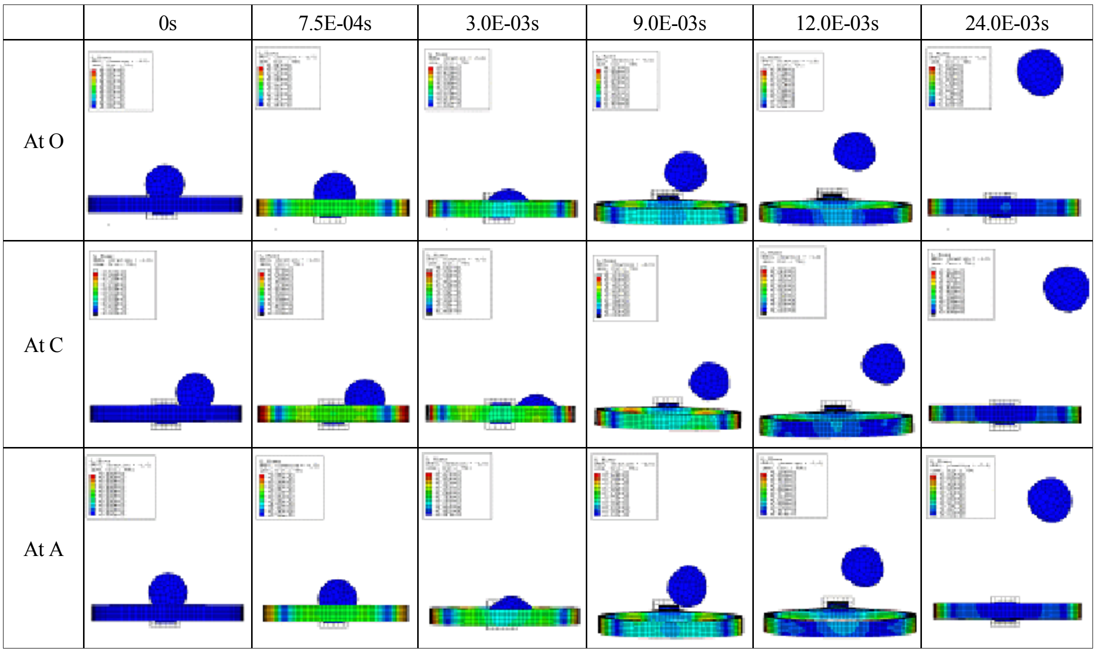


Fig. 8. Snapshots of the ball and the racket at different times during impact at points O, C, and A, respectively.

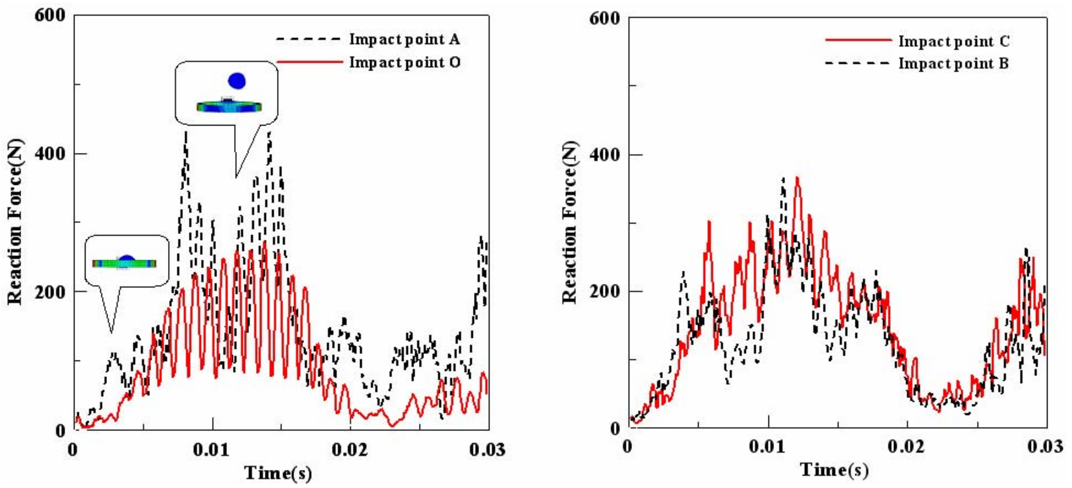


Fig. 9. Resultant reaction force on a hand during impact at different impact positions with string tension of 222N (Inserted figures show when the maximum reaction force occurred).

the resultant reaction force was evaluated at the reference point modeled with a rigid hand.

The resultant reaction forces on the rigid hand for different impact points were compared. As shown in Figs. 9(a) and 9(b), except when the ball hit the dead spot A of the racket, the player's hand felt the maximum resultant reaction force near t=0.012 second after the tennis ball left the racket. In this study, as the tennis ball hits the dead spot A, the player's hand feels a maximum resultant force of 424N, which is 1.61 times higher than the force it feels when the ball hits the sweet spot O, at t=0.081 second and t=0.0149 second. This indicated a severe vibration felt by the player's hand.

The ball's impact at the best-bounce spot B and the off-center spot C has a maximum resultant force of 366N, which is also higher than the maximum resultant force of 262N when the ball hits the sweet spot O.

Table 3. Vibration modes for the free boundary condition, fixed boundary condition, and hand grasp boundary condition.

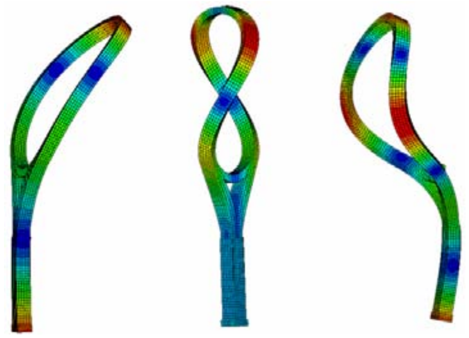
Mode	Free B.C	Fixed B.C	Hand grasp B.C
1		33.0	42.4
2	0	34.1	44.8
3	0	105.8	120.4$^{(1)}$
4	0	158.7$^{(1)}$	205.3
5	0	174.8	222.1
6	0	221.9	233.4
7	162.3$^{(1)}$	386.0	477.3$^{(2)}$
8	172.8	428.6	487.1
9	243.6	457.2$^{(2)}$	527.9$^{(3)}$
10	396.4	599.9	600.5
11	403.1$^{(2)}$	758.6	856.8
12	427.4$^{(3)}$	909.9$^{(3)}$	974.1
13	608.8	969.3	
14	757.1		
15	897.8		
16	993.6		

Moreover, the impulse transmitted through the racket to the hand elapses rapidly within 0.02s when the ball hits the best-bounce spot B. Results of this study indicated that the rebound force felt by the hand is the weakest, thereby resulting in the lowest possibility of injury, when the ball hits the racket at the sweet spot.

Meanwhile, this study also found out that the resultant moment (twisting) of the hand varies depending on the distance of the lever arm from the reference point of the rigid hand. This meant that there is less twisting of the hand if the ball hits the racket at the sweet spot. Conversely, the hand feels more force and requires more twisting, thereby raising the possibility of tennis elbow, if the ball hits a spot other than the sweet spot.

3.3 Vibration analysis

The modes of vibration of a racket depend on how the racket is held or clamped. Table 3 shows that the different boundary conditions yield distinct vibration modes. The bending mode for the free boundary condition at 162.3 Hz is the same as that for the fixed boundary condition at 158.7 Hz and the hand grasp boundary condition at 120.4Hz. This indicates that as the boundary condition of the grip is fixed, the frequency becomes smaller than the free boundary condition.



(a) 162.3Hz (b) 403.1Hz (c) 421.4Hz

Fig. 10. Vibration modes of (a) bending, (b) torsion, and (c) saddle for free boundary condition.

Fig. 10 shows the typical vibration modes for the free boundary condition: (a) bending mode at 162.3Hz, (b) torsion mode at 403.1Hz, and (c) saddle mode at 421.4Hz. In designing a tennis racket, achieving the lowest bending mode of vibration, which has the largest amplitude and involves the most energy, should be an objective.

4. Conclusion

A finite element simulation of the impact of a tennis ball on a tennis racket revealed that the string tension affected both the ball rebound speed and the accuracy. Within the recommended tension range, a lower tension provided more power and less impact on the arm, while a higher tension offered more control. Therefore, selecting the right string tension and impact point control are the most important elements in playing tennis.

In addition, hitting the ball at the dead spot of the racket was found to produce a higher torque and exerted a more rebound force on the hand, thereby increasing the tennis player's risk of having tennis elbow. In detail, as a tennis ball hits the dead spot A, the players' hand feels a maximum resultant force of nearly 424N, which is 1.6 times higher than the force felt if the ball hits the sweet spot, at t=0.081 second and t=0.0149 second. Moreover, the ball hitting the best bounce spot B and the off-center spot C also resulted in a force that was 1.4 times higher than if the ball hit the sweet spot O. Although the FEM simulation conducted for this study used only one type of racket, one type of string, and three string tensions, the results are valuable in quantitatively understanding the effects of string tension and the bearing of the impact spot on tennis performance, and on reducing or increasing one's chances of getting an elbow injury.

Acknowledgments

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